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COMPUTATION OF TRANSMISSION LINE TRANSIENTS BY USING FAST INVERSE LAPLACE TRANSFORM

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Abstract: In this paper, transients appearing in power transmission systems due to line energization are computed by using Fast Inverse Laplace Transform (FILT). s-domain transfer function of the system is calculated by using distributed parameter representation of transmission line. For the frequency to time domain transformation of transfer function, Fast Inverse Laplace Transform is applied. Comparisons are made between the results obtained by FILT and conventional methods such as conventional Laplace transform and Electromagnetic Transients Program (EMTP).

1. INTRODUCTION

Transmission line is one of the most important parts of the electric power system. The function of the transmission line is to transmit power between generating stations and main load centers. Over the years, electrical energy is usually transmitted at 245, 345, 400 or 500 kV. With the increasing consumption of electricity the power transmitted increases and for this to be achieved economically transmission voltages must rise. Nowadays transmission systems with line to line voltages of 750 or 1150 kV are being used.

Insulation requirements of a system is mostly determined by the level of transients initiated due to switching operations such as closing of circuit breakers. The peak values and duration of these transients must also be known for the design of protective devices.

Various time or frequency domain methods are suggested and used for computation of power network transients [1-4] In Fourier or Laplace transform methods the transformations are infinite integrals [2,3], and the inverse transform is an integral with respect to frequency ω or $s=j\omega$. In practice, these infinite integrals are evaluated numerically as finite summations. Summations are performed for $-\omega_{max}<\omega<\omega_{max}$ in steps of sampling frequency $\Delta\omega$. For best accuracy, ω_{max} should be large and $\Delta\omega$ small; in this case computer time may be very large. Therefore, in order to speed up the calculations fast transform techniques must be used.

In this paper, the application of FILT [5] for the computation of power system transients is presented. The method, which is initially developed by Hosono, is fast and efficient for the frequency to time domain conversion. The results obtained are compared by those obtained by conventional Laplace transform and Electromagnetic Transients Program (EMTP) in both accuracy and CPU times point of views.

2. TRANSMISSION LINE EQUATIONS

In transform techniques, the jω- or s-domain expression for transient voltage at any point in the network is derived. It is generally impossible to obtain a closed-form time-domain solution for a general input signal waveform. Therefore, a numerical Fourier or Laplace transform method is used.

Whether the transmission line is overhead or underground, its four electrical characteristics R′, L′, G′ and C′ (total resistance, inductance, conductance and capacitance, respectively) are distributed along the line. Voltage and current at any point on the line can be represented by both space and time dependent partial differential equations.

Let us consider the differential length of line shown in Fig. 1. The voltage and current wave propagations along the line (at a point x) are related to the line's distributed resistance (per unit length) R, inductance L, conductance G and capacitance C, by the equations

$$-\frac{dv}{dx} = zi, \qquad (1a)$$

$$-\frac{di}{dx} = yv, \qquad (1b)$$

where z=R+sL and y=G+sC are the series impedance and shunt admittance per unit distance, and v and i are the voltage and current phasors in the conductor. Differentiating equations (1.a) and (1.b) again with respect to x gives

$$\frac{d^2v}{dx^2} = zyv, \tag{2a}$$

$$\frac{d^2i}{dx^2} = yzI. \tag{2b}$$

The solution for terminal voltages and currents of the line with length l can be obtained in frequency domain as [6]

$$V_S = V_R \cosh \gamma l + I_R Z_0 \sinh \gamma l, \tag{3a}$$

$$I_S = I_R \cosh \gamma l + V_R Z_0^{-1} \sinh \gamma l, \tag{3b}$$

where the subscripts R and S stand for the receiving- and sending-ends, respectively. Z_0 and γ in (3) are complex characteristic impedance and propagation constant, respectively; they are defined by

$$Z_0 = \sqrt{z/y}, \tag{4a}$$

$$\gamma = \sqrt{zy}. \tag{4b}$$

In the case of multi-phase transmission lines series impedance z and shunt admittance y in (1.8) are replaced by series impedance matrix Z and shunt admittance matrix Y. In the solution procedure these equations are decoupled by applying modal analysis techniques [7] and solved in the same manner as single-phase case:

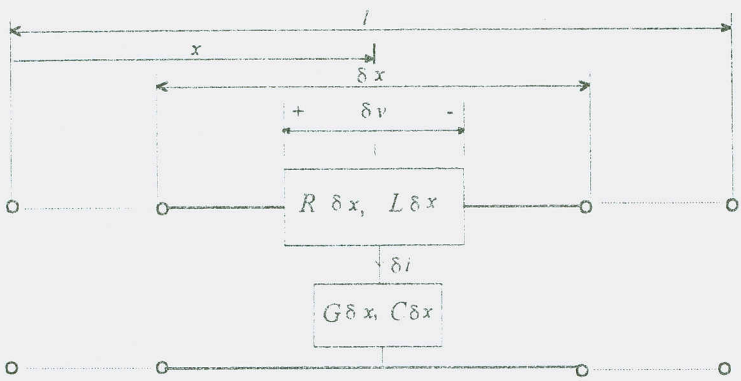


Figure. 1. A differential length of transmission line.

3. FAST INVERSE LAPLACE TRANSFORM

Fast inverse Laplace transform which is originally developed by Hosono [5] is used for frequency to time domain conversion. The method is based on evaluating the time function *f(t)*, corresponding to *F(s)*, from the series

$$f_{ec}^{kp}(t,a) = (e^{a}/t)\left[\sum_{n=1}^{k-1} F_n + (1/2^{p-1})\sum_{n=0}^{p} A_{pn}E_{k+n}\right], \qquad (5)$$

where,

$$F_n = (-1)^n \operatorname{Im} F\{[a + j(n-0.5)\pi]/t\}, \qquad (6)$$

with a>>1, and the coefficients A_{pn} are defined recursively by

$$A_{pp} = 1, \quad A_{pn-1} = A_{pn} + \binom{p+1}{n}. \qquad (7)$$

Many algorithms for the numerical computation of the inverse Laplace transform are found in literature. The algorithm developed by Hosono has been selected for its accuracy, efficiency, and ease of implementation. Details about FILT can be found in Reference [5].

4. APPLICATIONS AND RESULTS

Example 1- As an application of the method, a 160 km long, 400 kV transmission line with termination 50 mH inductance at receiving-end is considered. It is assumed that the line is energized by a step voltage and the receiving-end transient voltages due to line energization obtained by applying FILT, by taking the parameter in (5) as a=6, k=50 and p=20.

Receiving-end voltages are also calculated by the conventional Laplace transform. For this method, computation is carried in the range $0 \leq \omega \leq \omega max = 200$ rad/s with sampling rate $\Delta\omega = 50$ rad/s. Comparison of results obtained by FILT and conventional Laplace transform is shown in Fig. 2, and computer CPU times spent by two methods are shown in Table 1. FILT is approximately 15 times faster in CPU time.

Table 1. CPU times for step response of a single-phase line.

Method	CPU times
Conventional Laplace technique	2.56 min.
FILT	12 sec.

Example 2- In this example, transient overvoltages due to three-phase transmission line energization are studied. The 380 kV, 271 km line [8] with open circuit at receiving-end is energized by the sinusoidal voltage source with series inductance L_S=50 mH. Phase-A of the sinusoidal source at the instant of energization is at its peak value.

Sending-end and receiving-end voltage variations of phases A, B and C for one cycle of input obtained by FILT are shown in Fig. 3a and b. For comparison, the same example is studied by EMTP and sending- and receiving-end voltage transients (of phase A) obtained by EMTP and FILT are shown in Fig. 4a and b. These two figures show that the voltage curves obtained by EMTP and FILT are almost the same. Computer CPU times for obtaining the results for two methods are shown in Table 2.

Table 2. Execution times for line energization.

Method	CPU times (s)
EMTP	13.0
FILT	10.3

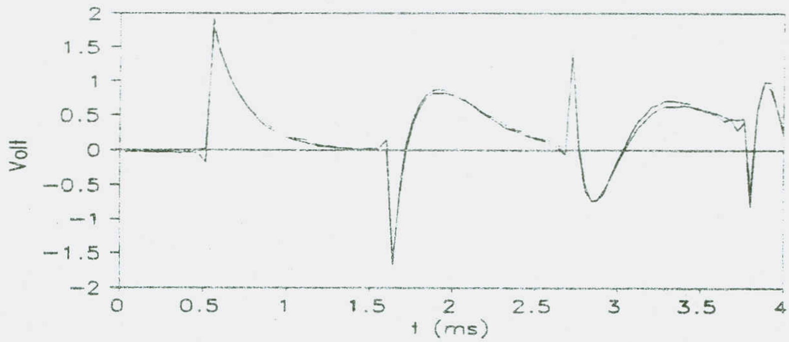
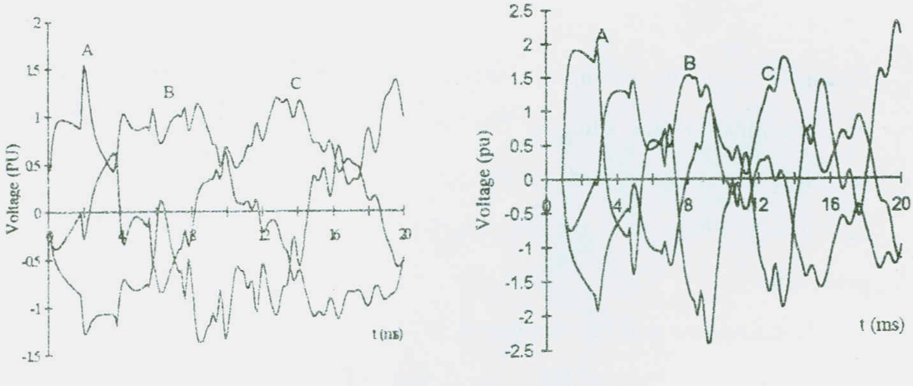
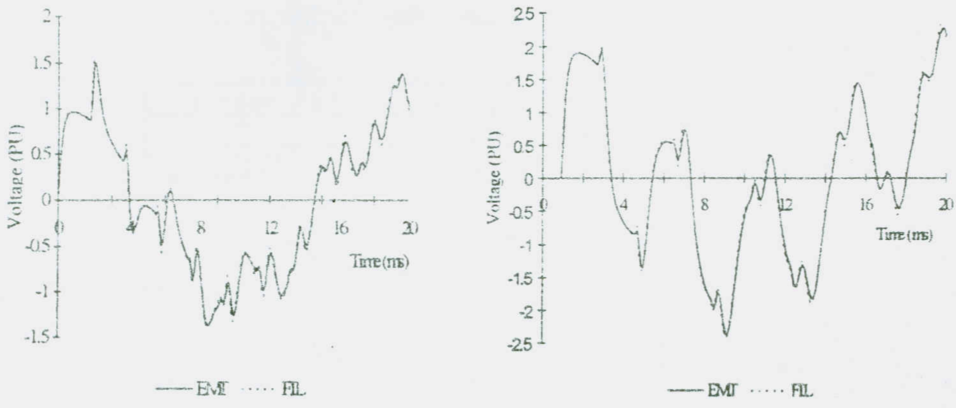


Figure 2. Step response of a single-phase line with 50 mH termination.



(a) (b)

Figure 4. Transients due to three-phase simultaneous energization; a) sending-end voltages, b) receiving-end voltages.



(a) (b)

Figure 5. Comparison of EMTP and FILT results; a) sending-end, b) receiving-end voltages for phase-A.

5. CONCLUSIONS

Single- and three-phase transmission line transients are computed by using FILT. The validation of the FILT was accomplished by comparing its results with those obtained by conventional Laplace transform and EMTP. The results show that the technique used is accurate and easy to implement on the digital computer. The method is considered to be extended to include frequency dependent parameters.

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