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Sliding mode controller design with fractional order differentiation: applications for unstable time delay systems

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Abstract: This paper presents a design method for a sliding mode controller with the contribution of a fractional order differential operator. The conventional sliding mode controller has been widely studied in different control applications. This paper proposes that the fractional order differential operator enlarges the output span of the classical sliding mode controller to obtain a better-fitting control signal for enhanced control performance. The sliding surface and the equivalent control law are modified with the addition of a fractional differential operator and a conventional one. The proposed sliding mode controller with fractional order differentiation is applied to the unstable time delay systems successfully. Illustrative examples are presented to demonstrate the performance of the proposed design method.

Key words: Sliding mode control, fractional order differentiation, unstable systems, time delay

1. Introduction

Fractional order systems and fractional order control structures are a promising research area in engineering problems. Recent research efforts have been focused on developing analysis and design procedures to extend classical control methods with fractional order integro-differential operators [1]. The first discussion of a fractional order derivative can be traced back to the 1690s, when L'Hospital wrote to Leibniz speculating about whether the order of a derivative could become a noninteger [2]. In the last several decades, with a better theoretical understanding of fractional calculus and subsequent developments in computing technologies, fractional calculus has begun to be widely utilized in various science and engineering areas [3]. The implementations of fractional order integro-differential operators have brought new horizons in control engineering. Consequently, fractional order integro-differential operators have been used in many control system applications in recent studies [4–7].

Some recent research efforts appear to use fractional integro-differential operators in sliding mode control (SMC) structures for different control systems' design applications. The conventional SMC structure is one of the well-known topics in control theory. The idea of using variable structure control for the control of a nonlinear system was proposed by Emelyanov [8]. SMC is a variable structure control system that increases the robustness of the system and preserves stability under parameter fluctuations. Much research has been conducted for the application of conventional SMC in recent studies. For instance, SMC adaptation and chattering reduction techniques for DC motor drives were proposed in [9]. A sliding mode observer was used in conjunction with a disturbance observer to predict the states of a slave system in [10]. SMC design methodology for a class of

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uncertain switched hybrid systems with unmeasurable states was given in [11]. The research efforts on combining the SMC structure with fractional order differentiation (FOD) bring a new perspective to this control technique. Vinagre and Calderon proposed a controller with the fractional order control law and sliding surface for the double integrator structure [12]. Valério and Costa proposed a fractional order SMC (FOSMC) structure for liquid flow in a 3-tank system [13]. Huang et al. used FOSMC for synchronous motor position control [14]. Batalov et al. applied this structure to a 3 degrees of freedom robot system driven by DC motors [15]. Dadras and Momeni controlled a nonlinear uncertain system via FOSMC [16]. Efe proposed the FOSMC with the reaching law approach in [17]. Consequently, the modification of a sliding mode controller for an unstable time delay system, in which the span of control output is enlarged with the fractional order differential operator, will contribute to research in this field.

A time delay is unavoidable in the practice of real control systems due to measurement lags, analysis times, computation lags, etc. [18]. A control system designer may sometimes neglect a relatively small delay in which the system still satisfies the design requirements, but a time delay frequently causes instability in real applications and it cannot be underestimated. Therefore, the stability problems of this kind of control system have been a main research subject for many researchers in the recent decades [19]. Additionally, when a plant has an unstable pole, the effects of time delay have to be considered along with the impact of the unstable pole on the stability of the system [20]. Thus, the SMC is an appropriate technique to overcome this undesirable situation. There have been several examples to overcome the stabilization problem with this controller, such as the combined approach of predictive structures with SMC for some long time delay systems investigated in [21] and the SMC for unstable first order plus delay time processes designed in [22].

This paper proposes to use the fractional order differential operator together with the integer order one to compute the sliding surface and equivalent control law of the conventional SMC. The proposed method is illustrated for unstable systems with time delay. It is clear from the simulation results that the proposed structure of the fractional differentiation enhances the control efforts of the conventional SMC, which results in better control performance.

The paper is organized as follows. Section 2 presents the design of the sliding mode controller with FOD (SMC-FOD). In Section 3, simulation results are presented demonstrating the control performance for unstable systems. Section 4 is devoted to the conclusions.

2. Sliding mode controller with FOD

2.1. Motivation

A conventional SMC, which is derived from variable structure control, is widely used to control nonlinear systems and preserve stability under parameter fluctuations [23]. Additionally, this control structure is independent of model uncertainties and disturbances. The main objective of SMC is to determine the controller signal that forces the system's error along a sliding surface. Thus, the SMC approach provides stable and robust control performance, despite the effect of external disturbances and model uncertainties, by staying confined to the sliding surface. This robust controller structure involves 2 steps. The first step is the selection of a stable plane, called the switching function or sliding surface $S(t)$, and the second step is to determine the control law for reaching and staying on the sliding surface. Consider the following negative unity feedback system given in Figure 1.

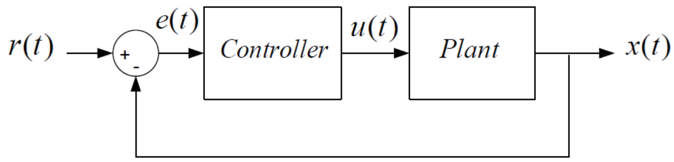

Figure 1. Block diagram of the closed-loop system.

The sliding surface $S(t)$ can be defined as follows [24]:

$$S(\bar{x};t) \;=\; \left(\frac{d}{dt} \,+\, \lambda\right)^{n\,-\,1} e(t), \tag{1}$$

where $e(t) = x_d(t) - x(t)$ is the tracking error of state variable $x(t)$, $x_d(t)$ is the desired trajectory, λ represents the time constant, and n is the order of the system to be controlled. As seen from Eq. (1), the order of the switching function is less than the order of the plant. The control objective is to keep the system on the sliding surface as given in Eq. (1). The trajectory of the tracking error reaches the switching line, $S(t) = 0$, and then slides along it towards the origin. In order to determine the control law, the Lyapunov function approach is defined as follows:

$$V(S) = \frac{1}{2}S^2(t). \tag{2}$$

The system reaches $S(t) = 0$ in finite time if the above Lyapunov function satisfies

$$\frac{1}{2}\frac{d}{dt}S^2(t) \;\leq\; -\eta\,|S(t)|\,, \tag{3}$$

for positive constant η. Eq. (3) gives a condition to reach $S(t) = 0$, which is called the reaching law. Using Eq. (3), one can obtain the following equation to force $e(t)$ to be 0 at all times:

$$S\dot{S} \;\leq\; 0. \tag{4}$$

In the conventional SMC law $u(t)$ consists of 2 parts. The first is a discontinuous or corrective control law, which compensates for the deviations to reach the sliding surface. The second is a continuous or equivalent control law, which makes the derivative of the sliding surface equal to 0 to stay on the sliding surface. The control law is obtained as follows:

$$u(t) \;\;=\;\; u_{eq}(t) \;+\; u_d(t). \tag{5}$$

This paper proposes that the differentiation in the sliding surface $S(\bar{x};t)$ in Eq. (1) can be used together with the fractional order differential operator to enhance the effect of the differentiation. The developments in the solution methods for the fractional order calculus enable us to use the FOD easily. Fractional calculus can be considered to be the generalization of the integration and differentiation of the integer order expressions to the noninteger order one. The most frequently used integro-differential definitions are those given by Grünwald and Letnikov, Riemann and Liouville, and Caputo. The general form of a fractional order integro-differential equation can be defined as follows [25,26]:

$$ {}_aD_t^{\alpha} = \begin{cases} \frac{d^{\alpha}}{dt^{\alpha}} & Re(\alpha) > 0 \\ 1 & Re(\alpha) = 0 \\ \int\limits_a^t (d\tau)^{-\alpha} & Re(\alpha) < 0 \end{cases}, \tag{6}$$

where α is the order of the differentiation or integration. The most general formula for the Laplace transformations of the integro-differential expressions can be given as follows:

$$L\left\{\frac{d^m f(t)}{dt^m}\right\} = s^m L\left\{f(t)\right\} - \sum_{k=0}^{n-1} s^k \left[\frac{d^{m-1-k} f(t)}{dt^{m-1-k}}\right]_{t=0}, \tag{7}$$

where n is an integer number and m satisfies $n-1 < m < n$ [26].

The motivation of the present paper comes from understanding the possibility of improving the control effort of the sliding mode controller structure using fractional differentiation for computation of control law and sliding surface. There are some fractional order sliding mode controller methods in recent studies. However, this study proposes to use small deviations in FOD to enlarge the span of the control effort against the output disturbances.

2.2. Problem formulation

Let us denote $(d/dt) = D$ in Eq. (1) for the nth order system as follows:

$$S(\bar{x};t) \;=\; (D+\lambda)^{n-1} e(t). \tag{8}$$

The sliding surface $S(t)$ in Eq. (8) can be written using a binomial expansion as:

$$S(\bar{x};t) = \left[\sum_{k=0}^{i} \begin{pmatrix} i \\ k \end{pmatrix} [D]^k \lambda^{i-k}\right] e(t), \tag{9}$$

where $i = n-1, \quad k = 0, 1, \cdots n-1$.

Remark 1: The fractional differentiation D^α enlarges the effect of differentiation on the sliding surface in Eq. (9) as follows:

$$S(\bar{x};t) = \left[\sum_{k=0}^{i} \begin{pmatrix} i \\ k \end{pmatrix} \left[(D)^k D^{\pm\alpha_k}\right] \lambda^{i-k}\right] e(t). \tag{10}$$

2.3. Controller design

Consider the following transfer function:

$$G(s) = \frac{K}{b_n s^n + b_{n-1} s^{n-1} + \ldots + b_1 s + b_0}. \tag{11}$$

The sliding surface and derivative of the sliding surface with the fractional order differential operator can be obtained for this transfer function using Remark 1 as follows:

$$S(t) \;=\; n_k D^{k\pm\alpha_k} e(t) \;+\; n_{k-1} D^{k-1\pm\alpha_{k-1}} e(t) \;+\; \cdots \;+\; n_1 D^{1\pm\alpha_1} e(t) \;+\; n_0 D^{0\pm\alpha_0} e(t), \tag{12}$$

$$\dot{S}(t) \;=\; n_k D^{k+1\pm\alpha_k} e(t) \;+\; n_{k-1} D^{k\pm\alpha_{k-1}} e(t) \;+\; \cdots \;+\; n_1 D^{2\pm\alpha_1} e(t) \;+\; n_0 D^{1\pm\alpha_0} e(t), \tag{13}$$

where $n_k, \; n_{k-1}, \; \cdots n_1, \; n_0$ are the coefficients obtained from the binomial expansion of Eq. (10). One can compute from the binomial expansion that $n_k = 1$. $\alpha_k, \; \alpha_{k-1}, \; \cdots \alpha_1, \; \alpha_0$ are real numbers in the interval

$0 < \alpha_k < 1$. Eq. (3) states that the system reaches $S(t) = 0$ in finite time. Consider $\dot{S}(t) = 0$ in Eq. (13) to define the equivalent control law as follows:

$$D^{k+1}D^{\pm\alpha_k}e(t) + n_{k-1}D^{k\pm\alpha_{k-1}}e(t) + n_{k-2}D^{k-1\pm\alpha_{k-2}}e(t) + \cdots\cdots + n_0D^{1\pm\alpha_0}e(t) = 0, \quad (14)$$

$$D^{k+1}e(t) = -n_{k-1}D^{k\pm\beta_{k-1}}e(t) - n_{k-2}D^{k-1\pm\beta_{k-2}}e(t) - \cdots\cdots - n_0D^{1\pm\beta_0}e(t), \quad (15)$$

where $\beta_{k-1}, \beta_{k-2}, \cdots, \beta_0$ are the fractional orders of derivatives that are obtained by dividing $D^{\pm\alpha_k}$. One can obtain the following equation, using Eq. (11) in Figure 1, and define it in the time domain, while the initial conditions are assumed to be 0:

$$u(t) = b_mD^{k+1}x(t) + b_{m-1}D^kx(t) + \cdots\cdots + b_1D^1x(t) + b_0x(t), \quad (16)$$

where $u(t)$ is the control signal, n is the order of the system for $k + 1 = n$, and $b_m = b_n/K$.

The equivalent control law can be determined as follows using Eq. (15) in Eq. (16):

$$\begin{array}{c} u_{eq}(t) = b_m(-n_{k-1}D^{k\pm\beta_{k-1}}x(t) - n_{k-2}D^{k-1\pm\beta_{k-2}}x(t) - \cdots\cdots - n_0D^{1\pm\beta_0}x(t)) + \\ b_{m-1}D^kx(t) + b_{m-2}D^{k-1}x(t) + \cdots\cdots + b_1D^1x(t) + b_0x(t) \end{array}, \quad (17)$$

for $e(t) = x_d(t) - x(t)$ and $x_d(t) = 0$. In order to satisfy the reaching condition under such uncertainties and disturbances, a term, which is discontinuous across the line $S = 0$, is added to $u_{eq}(t)$. Next, the total control law becomes as follows:

$$\begin{array}{c} u(t) = b_m(-n_{k-1}D^{k\pm\beta_{k-1}}x(t) - n_{k-2}D^{k-1\pm\beta_{k-2}}x(t) - \cdots\cdots - n_0D^{1\pm\beta_0}x(t)) + \\ b_{m-1}D^kx(t) + b_{m-2}D^{k-1}x(t) + \cdots\cdots + b_1D^1x(t) + b_0x(t) - K_d sign(S(t)) \end{array}, \quad (18)$$

where the sign function is defined as

$$sign(S(t)) = \begin{cases} 1 & S(t) > 0 \\ -1 & S(t) < 0 \end{cases}. \quad (19)$$

One can obtain the block diagram in Figure 2 for the SMC-FOD structure using Eq. (18).

It is important to select the parameters K_d and λ, which should satisfy the reaching condition in the presence of model uncertainties and external disturbances. It is impossible to achieve instantaneous high switching because of the physical limitations that cause chattering. Chattering is a high-frequency oscillation that is undesirable in practice because of its high control activity [24]. There are several methods to overcome the problem of chattering. In this paper, the signum function is replaced with the saturation function, which can be written as follows:

$$sat(S(t)) = \begin{cases} sign(S(t)) & |S(t)| > \phi \\ \frac{S(t)}{\phi} & |S(t)| \le \phi \end{cases}, \quad (20)$$

where ϕ is the boundary layer. The amplitude of chattering is directly proportional to parameter K_d. Additionally, the performance of the system is sensitive to bandwidth λ.

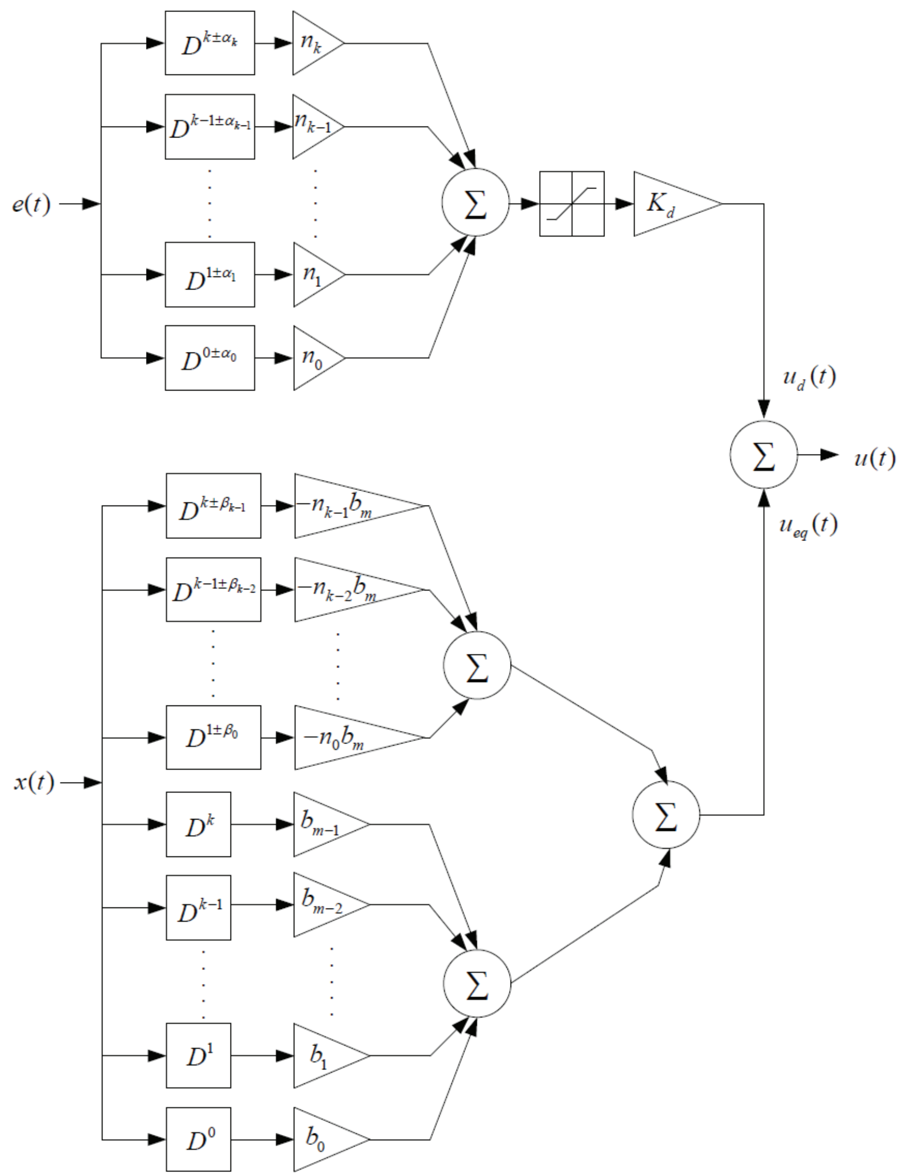

Figure 2. Block diagram of the proposed SMC-FOD.

2.4. Stability analysis of SMC-FOD with the saturation function

Some new results on the fractional SMC approach use sign and saturation functions for stability analysis [27]. In this manuscript, the saturation function is used for the stability analysis of the proposed SMC-FOD. Consider the transfer function in Eq. (11), derivative of the sliding surface in Eq. (13), and total fractional control law in Eq. (18). One can use the saturation function in the control law in Eq. (18) as follows:

$$\begin{aligned} u(t) \; = \; & b_m(-\, n_{k-1}D^{k\,\pm\,\beta_{k\,-\,1}}x(t) \;-\; n_{k\,-\,2}D^{k\,-\,1\,\pm\,\beta_{k\,-\,2}}x(t) \;-\; \cdots\cdots \;-\; n_0D^{1\,\pm\,\beta_0}x(t)) \;+ \\ & b_{m\,-\,1}D^kx(t) \;+\; b_{m\,-\,2}D^{k\,-\,1}x(t) \;+\; \cdots\cdots \;+\; b_1D^1x(t) \;+\; b_0x(t) \;-\; K_d sat(S(t)) \end{aligned} \quad . \tag{21}$$

Substituting Eqs. (21) and (11) into Eq. (13), we have:

$$\begin{aligned}\dot{S}(t) = n_k D^{\pm\alpha_k}\left[(-n_{k-1}D^{k\pm\beta_{k-1}}x(t) - n_{k-2}D^{k-1\pm\beta_{k-2}}x(t) - \cdots - n_0 D^{1\pm\beta_0}x(t)) - \tfrac{K_d}{b_m}sat(S(t))\right] \\ +n_{k-1}D^{k\pm\alpha_{k-1}}x(t) + \cdots + n_1 D^{2\pm\alpha_1}x(t) + n_0 D^{1\pm\alpha_0}x(t)\end{aligned}, \quad (22)$$

where $\beta_{k-1} = \alpha_{k-1} \mp \alpha_k$ and $n_k = 1$, and Eq. (22) can be rewritten as follows:

$$\begin{aligned}\dot{S}(t) \ = \Big[- n_{k-1}D^{k \pm \alpha_{k-1}}x(t) - n_{k-2}D^{k-1 \pm \alpha_{k-2}}x(t) - \cdots - n_0 D^{1 \pm \alpha_0}x(t) - D^{\pm \alpha_k}\tfrac{K}{b_n}K_d sat(S(t))\Big] \\ + n_{k-1}D^{k \pm \alpha_{k-1}}x(t) + \cdots + n_1 D^{2 \pm \alpha_1}x(t) + n_0 D^{1 \pm \alpha_0}x(t)\end{aligned}. \quad (23)$$

After the simplifications, the following equation can be obtained:

$$\dot{S}(t) \ = D^{\pm \alpha_k}\left[-\frac{K}{b_n}K_d sat(S(t))\right]. \quad (24)$$

Outside of the boundary layer, if $|S(t)| > \phi$, then $sat(S(t)) = sign(S(t))$ and we have the derivative of the Lyapunov function as follows:

$$\dot{V} = D^{\pm \alpha_k}\left[-\frac{K}{b_n}K_d sign(S(t))\right]S(t) < 0. \quad (25)$$

Therefore, the system is stable on the condition that $\frac{K}{b_n}K_d > 0$.

Inside the boundary layer, $|S(t)| \leq \phi$ and $sat(S(t)) = S(t)/\phi$. The derivative of the Lyapunov function is written as follows:

$$\dot{V} = D^{\pm \alpha_k}\left[-\frac{K}{b_n}K_d\frac{S(t)}{\phi}\right]S(t) < 0. \quad (26)$$

It is clear that $V > 0$ and $\dot{V} < 0$ for $\frac{K}{b_n}K_d > 0$. Therefore, the system with the SMC-FOD controller in the presence of the saturation function is stable. In order to improve the stability of the system, the effect of the fractional order integro-differential operator $D^{\pm \alpha_k}$ must be considered. A proper value of the fractional order of the differentiation α_k provides better control performance.

2.5. Application of the proposed SMC-FOD to unstable time delay systems

This paper is dedicated to time delay systems with one or more unstable poles. Both the unstable poles and time delay yield more complicated control processes and instability. Consequently, a suitable controller design for the unstable plant with time delay will be important. In this section, the stability problem of the transfer function with an unstable pole and time delay was studied. The unstable plant transfer function can be modeled as follows:

$$G(s) \ = \ \frac{Ke^{-ts}}{(\tau_n s \mp a_n)(\tau_{n-1}s \mp a_{n-1})\cdots(\tau_1 s \mp a_1)}, \quad (27)$$

where K, t, and τ_n are the gain, time delay, and time constant of the model, respectively. $a_n, a_{n-1}, \cdots, a_1$ are real numbers. A Padé approximation of the time delay for the SMC design is widely used in the literature. For example, a single-input single-output delay system tracking problem was considered in [28], with a second order SMC and Padé approximation. In [29], a higher order Padé approximation was used to construct a model of a transformed system without a time delayed output. In this paper, a Padé approximation is used to approximate the exponential expression by a polynomial function for these systems. Hence, the 0/1 Padé approximation is used to simplify the processes dynamics. According to the 0/1 Padé approximation for e^x,

$$\exp_{0/1}(x) = \frac{1}{1-x}. \tag{28}$$

Next, the time delay is defined as [30] follows:

$$e^{-ts} = \frac{1}{ts + 1}. \tag{29}$$

Thus, Eq. (27) becomes as follows:

$$G(s) = \frac{K}{(\tau_n s \mp a_n)(\tau_{n-1} s \mp a_{n-1}) \cdots (\tau_1 s \mp a_1)(ts + 1)}. \tag{30}$$

This transfer function can be written in the form of Eq. (11). Next, the SMC-FOD can be designed for an unstable time delay system, as given in Section 2.3.

3. Numerical examples and simulation results

In this section, the proposed fractional sliding mode controller is applied to unstable time delay systems in illustrative examples to demonstrate the performance of the control effort. Different processes under the effect of output disturbances were controlled by changing the order of the fractional differentiation operator D^α. The MATLAB/Simulink model of the SMC-FOD in Figure 2 is used to simulate the controller and the system. The sampling frequency is selected as 2 kHz. In the simulation, fourth order rational approximations of the fractional differentiation operator, s^α, $(0 < \alpha < 1)$, are obtained using the continuous fraction expansion (CFE) method, which is one of the most important approximations for fractional order systems. The CFE method can be expressed in the following form [31]:

$$(1 + x)^\alpha = \frac{1}{1-} \frac{\alpha x}{1+} \frac{(1+\alpha)x}{2+} \frac{(1-\alpha)x}{3+} \frac{(2+\alpha)x}{2+} \frac{(2-\alpha)x}{5 + ...}. \tag{31}$$

In Eq. (31), $x = s - 1$ is used for the computation of s^α [32].

Example 1 Consider a first order unstable plant with time delay, which is also given in [22], as follows:

$$G_1(s) = \frac{1}{s - 1} e^{-0.8s}. \tag{32}$$

This transfer function can be rewritten using Eq. (29) as follows:

$$G_1(s) = \frac{1}{(s - 1)(0.8s + 1)} = \frac{1}{0.8s^2 + 0.2s - 1}. \tag{33}$$

One can compute the sliding surface and equivalent control law using Remark 1 as follows:

$$S(t) = D^{1 \pm \alpha_1} e(t) + \lambda D^{0 \pm \alpha_0} e(t), \tag{34}$$

$$u_{eq}(t) \;=\; 0.8\,(\,-\,\lambda D^{1\,\pm\,\beta_0}x(t))\;+\;0.2\,(D^1x(t)) - x(t), \tag{35}$$

and the total control law is obtained as:

$$u(t) \;=\; 0.8(\,-\,\lambda D^{1\,\pm\,\beta_0}x(t))\;+\;0.2(D^1x(t))\;-\;x(t)\;-\;K_d sign(S(t)). \tag{36}$$

Different values of differentiation orders α_0, α_1, and β_0 in Eqs. (34)–(36) are used in the simulation models in Figure 2 to observe the effect of the order of fractional differentiation on the system performance. Figure 3 presents the step responses of the system in Eq. (33), controlled with SMC-FOD, to illustrate the effect of the parameters. The reference input is set to 1 and the saturation function is adopted in the control law. Figures 3a–3c present the effects of α_0, α_1, and β_0, respectively, while the other parameters remain 0. The best response to the SMC-FOD is obtained at values of $\alpha_0 = -0.01$, $\alpha_1 = -0.05$, and $\beta_0 = 0.1$. The proposed SMC-FOD method becomes a conventional SMC (CSMC) for the values of $\alpha_0 = 0$, $\alpha_1 = 0$, and $\beta_0 = 0$.

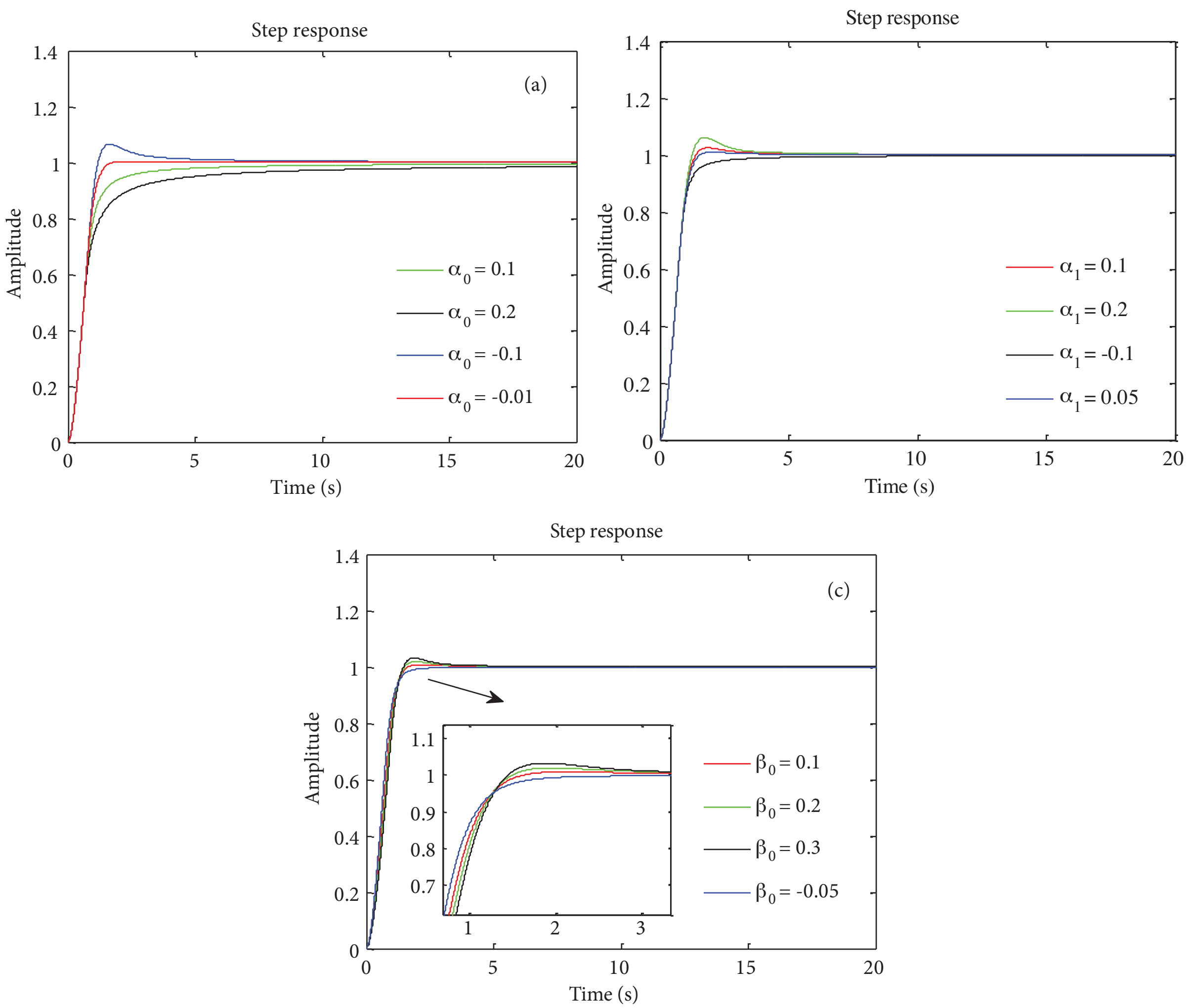

Figure 3. Step responses with different orders of differentiation: (a) step responses for different α_0 values, (b) step responses for different α_1 values, (c) step responses for different β_0 values.

Figures 4a and 4b present the step responses and control signal, respectively, for the CSMC and SMC-FOD. One can conclude from Figure 4 that the SMC-FOD provides better performance than the CSMC for the values of $\alpha_0 = -0.41$, $\alpha_1 = -0.4$, and $\beta_0 = 0.2$. The control signal that belongs to the proposed controller is appropriate. It is clear from Figure 5 that the control effort of the SMC-FOD with the same parameters compensates for the disturbance better than the CSMC.

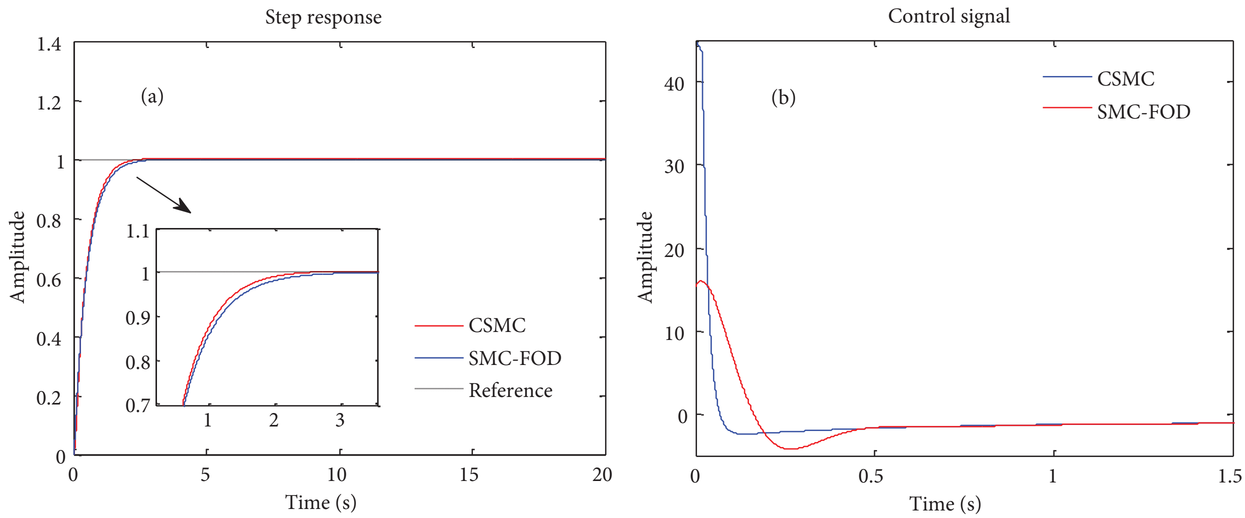

Figure 4. Step responses (a) and control signals (b) for $\alpha_0 = -0.41$, $\alpha_1 = -0.4$, $\beta_0 = 0.2$.

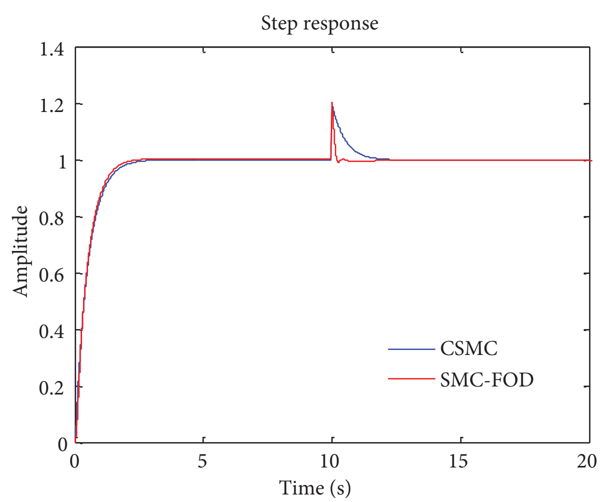

Figure 5. Step responses with output disturbance (noise) of 0.2 during 10–20 s ($\alpha_0 = -0.41$, $\alpha_1 = -0.4$, $\beta_0 = 0.2$).

In other words, the performance of the proposed SMC-FOD method is more robust than that of the CSMC method. The quality of the system responses can also be compared using the following integral absolute error (IAE) function.

$$IAE = \int |e| \, dt \tag{37}$$

The IAE is computed as $\Delta e = 0.6057$ for the CSMC and $\Delta e = 0.5172$ for the SMC-FOD, respectively. It can be seen that the performance of the SMC-FOD is better than that of the CSMC.

Example 2 Consider the following second order unstable plant with a time delay:

$$G_2(s) = \frac{1}{(0.5s + 1)(2s - 1)} e^{-0.5s}. \tag{38}$$

Eq. (38) can be rewritten using Eq. (29) in the form of Eq. (11), as follows:

$$G_2(s) = \frac{1}{0.5s^3 + 1.75s^2 + s - 1}. \tag{39}$$

One can compute the sliding surface and equivalent control law using Remark 1 as follows:

$$S(t) = D^{2 \pm \alpha_2} e(t) + 2\lambda D^{1 \pm \alpha_1} e(t) + \lambda^2 D^{0 \pm \alpha_0} e(t), \tag{40}$$

$$u_{eq}(t) = 0.5(-2\lambda D^{2 \pm \beta_1} x(t) - \lambda^2 D^{1 \pm \beta_0} x(t)) + 0.25 D^2 x(t) - 2D^1 x(t) - x(t), \tag{41}$$

and the total control law is obtained as:

$$\begin{array}{c} u(t) = 0.5(-2\lambda D^{2 \pm \beta_1} x(t) - \lambda^2 D^{1 \pm \beta_0} x(t)) + \\ 0.25 D^2 x(t) - 2D^1 x(t) - x(t) - K_d sign(S(t)) \end{array}. \tag{42}$$

Different values of differentiation orders α_0, α_1, α_2, β_0, and β_1 in Eqs. (40)–(42) are used in the block diagram of the SMC-FOD in Figure 2 to observe the effect of the order of fractional differentiation on the system performance. Figure 6 presents the step responses of the transfer function in Eq. (39) to illustrate the effect of the parameters in the SMC-FOD separately. Figures 6a–6e present the effects of α_0, α_1, α_2, β_0, and β_1, respectively, while the other fractional order parameters remain 0. The best response to the SMC-FOD is obtained at values of $\alpha_0 = -0.01$, $\alpha_1 = -0.01$, and $\alpha_2 = 0.1$ in Figures 6a–6c, respectively. The β parameters (especially β_1) in Figures 6d and 6e usually contribute in the presence of disturbing effects.

Figure 7 presents the control performance of the CSMC and SMC-FOD with parameters of $\alpha_0 = -0.01$, $\alpha_1 = 0$, $\alpha_2 = 0.04, \beta_0 = 0.7$, and $\beta_1 = 0.6$. One can see that the SMC-FOD compensates for the disturbance better than the CSMC. Additionally, the IAE is computed as $\Delta e = 1.1420$ for the CSMC and $\Delta e = 1.0082$ for the SMC-FOD. It can be seen that the performance of the SMC-FOD is better than that of the CSMC.

Example 3 Consider a third order unstable plant with a time delay as follows:

$$G_3(s) = \frac{1}{(s - 1)(0.2s + 1)(0.3s + 1)} e^{-0.1s}, \tag{43}$$

which was also given in [20]. This transfer function can be rewritten using Eq. (29) as:

$$G_3(s) = \frac{1}{0.006s^4 + 0.104s^3 + 0.49s^2 + 0.4s - 1}. \tag{44}$$

According to the conditions given by Remark 1, the equations for the proposed SMC-FOD controller for the above fourth order transfer function are obtained as follows:

$$S(t) = D^{3 \pm \alpha_3} e(t) + 3\lambda D^{2 \pm \alpha_2} e(t) + 3\lambda^2 D^{1 \pm \alpha_1} e(t) + \lambda^3 D^{0 \pm \alpha_0} e(t), \tag{45}$$

$$\begin{array}{c} u_{eq}(t) = 0.006(-3\lambda D^{3 \pm \beta_2} x(t) - 3\lambda^2 D^{2 \pm \beta_1} x(t) - \lambda^3 D^{1 \pm \beta_0} x(t)) + \\ 0.104 D^3 x(t) + 0.49 D^2 x(t) + 0.4 D^1 x(t) - x(t) \end{array}, \tag{46}$$

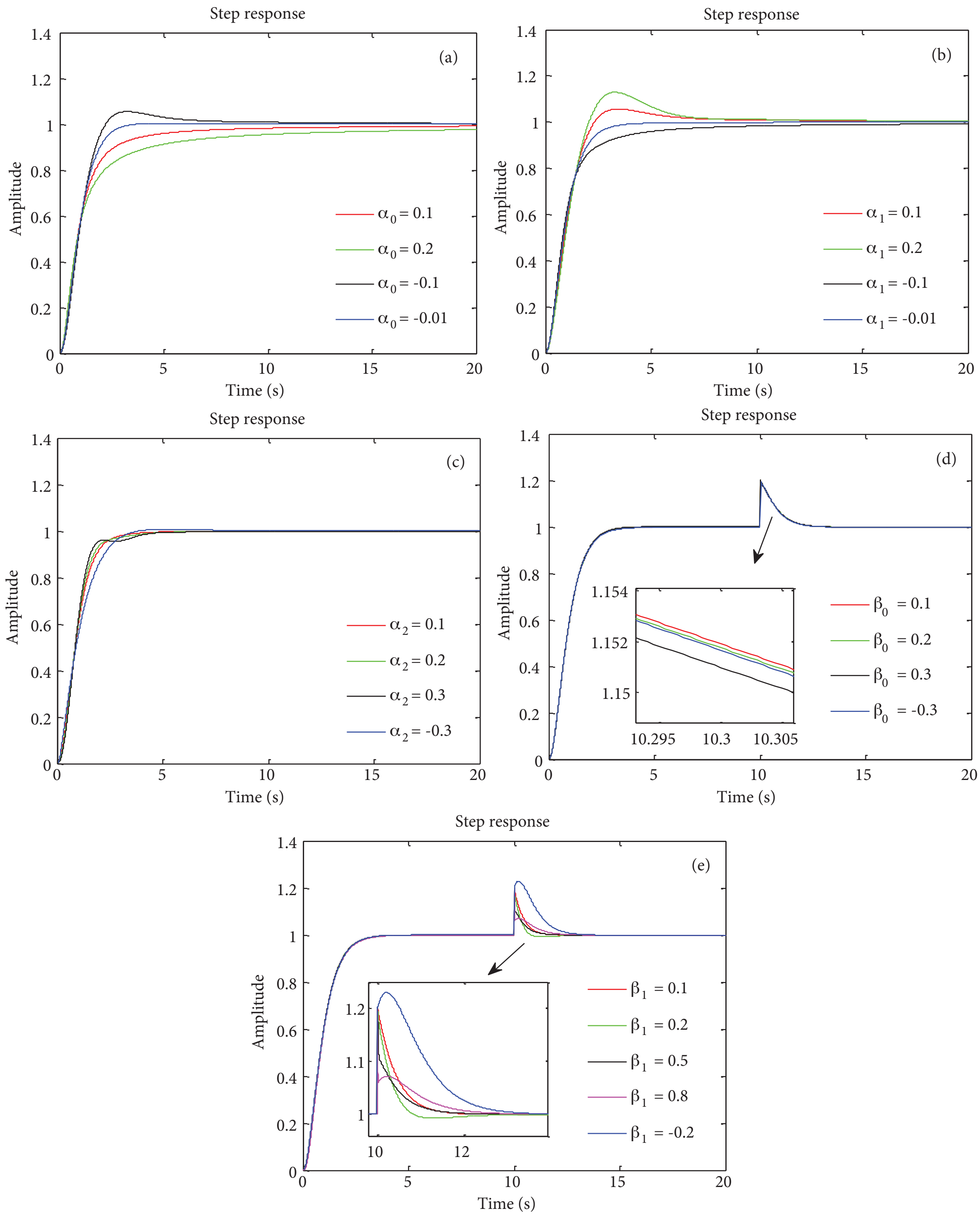

Figure 6. Step responses with different orders of differentiation: (a) step responses for different α_0 values, (b) step responses for different α_1 values, (c) step responses for different α_2 values, (d) step responses for different β_0 values, (e) step responses for different β_1 values.

and the total control law is obtained as:

$$\begin{array}{c} u(t) \;=\; 0.006(-3\lambda D^{3\pm\beta_2}x(t) \;-\; 3\lambda^2 D^{2\pm\beta_1}x(t) \;-\; \lambda^3 D^{1\pm\beta_0}x(t)) + \\ 0.104D^3x(t) \;+\; 0.49D^2x(t) \;+\; 0.4D^1x(t) \;-\; x(t) \;-\; K_d sign(S(t)) \end{array} \qquad (47)$$

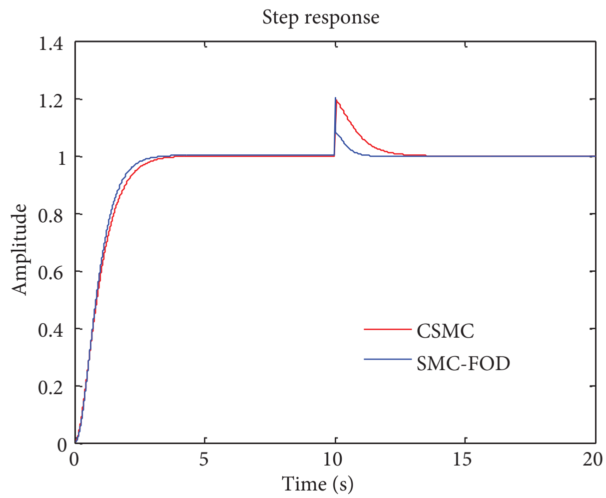

Figure 7. Step responses with output disturbance (noise) of 0.2 during 10-20 s.

Figure 8 presents the step responses of the transfer function in Eq. (43) to illustrate the effect of parameters α_0, α_1, α_2, α_3, β_0, β_1, and β_2 in the SMC-FOD separately. Figures 8a–8d present the effects of α_0, α_1, α_2, and α_3, and Figures 8e–8g present the effects of β_0, β_1, and β_2 separately, while the other parameters remain 0. The best response to the SMC-FOD is obtained at values of $\alpha_0 = -0.02$, $\alpha_1 = -0.002$, $\alpha_2 = -0.05$, $\alpha_3 = -0.1$, $\beta_0 = -0.1$, $\beta_1 = 0.1$, and $\beta_2 = 0.2$, respectively, in Figure 8. It is clear that the variation of the β parameters is more effective to compensate for the disturbances than the parameters of α. One can see

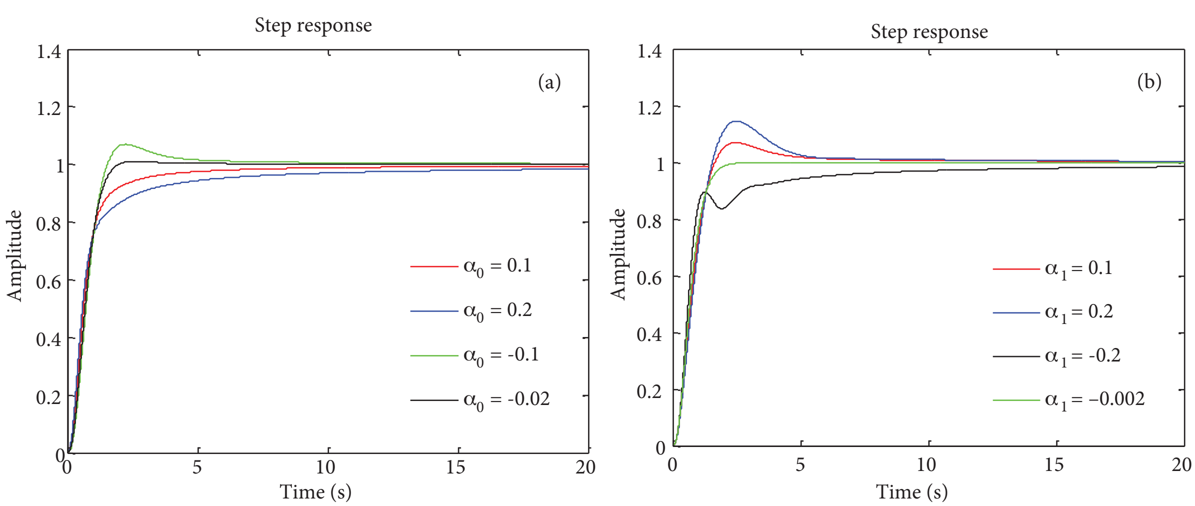

Figure 8. Step responses with different orders of differentiation: (a) step responses for different α_0 values, (b) step responses for different α_1 values.

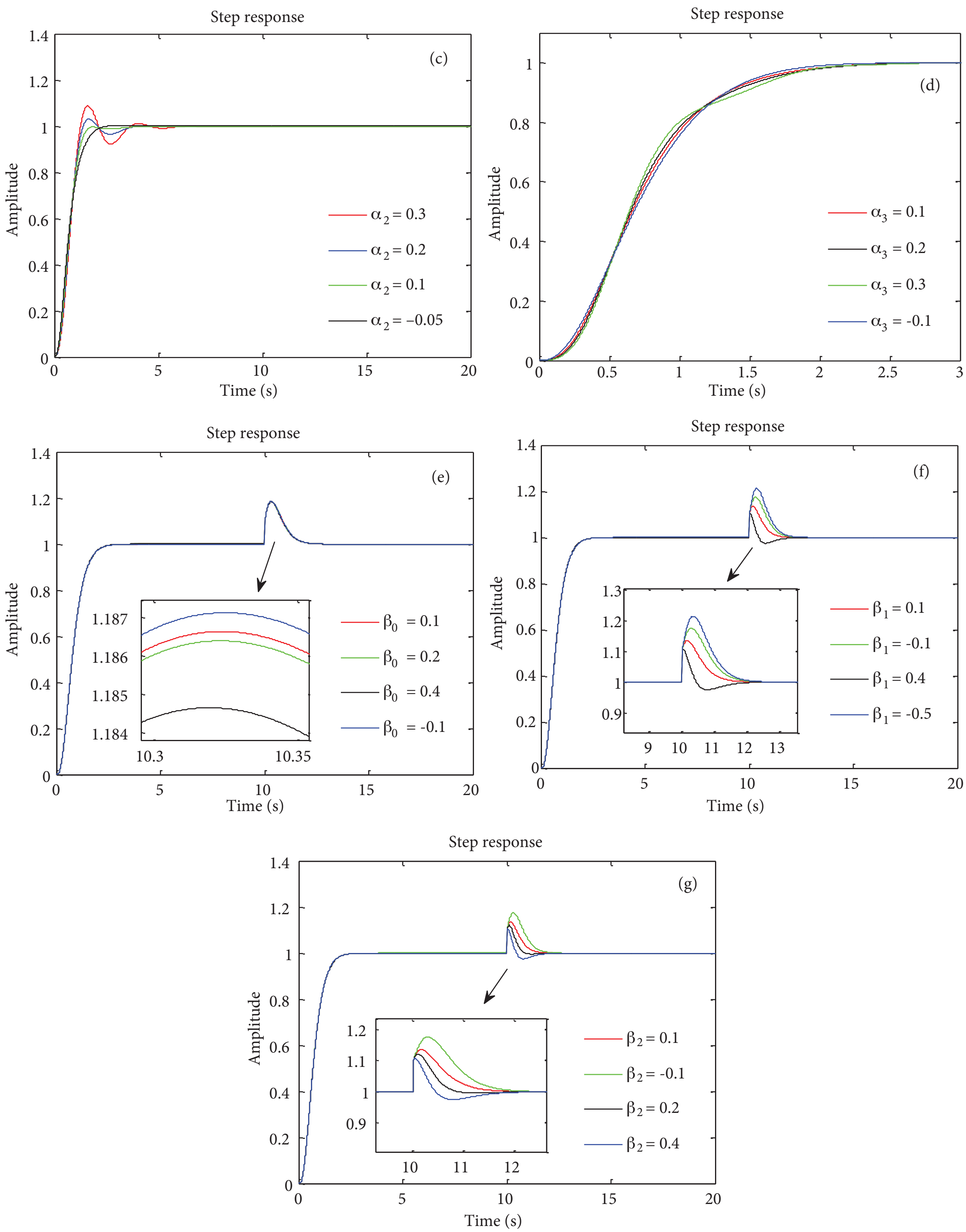

Figure 8. Step responses with different orders of differentiation: (c) step responses for different α_2 values, (d) step responses for different α_3 values, (e) step responses for different β_0 values, (f) step responses for different β_1 values, (g) step responses for different β_2 values.

from Figure 9 that when the output disturbance is added to the system at t = 10 s, the variation of the amplitude of the disturbance by the SMC-FOD method with the $\alpha_0 = -0.01$, $\alpha_1 = -0.02$, $\alpha_2 = -0.1$, $\alpha_3 = -0.5$, $\beta_0 = 0.8$, $\beta_1 = 0.8$, and $\beta_2 = 0.1$ parameters is smaller than that of the CSMC method. One can also compute the IAE as $\Delta e = 1.0316$ for the CSMC and $\Delta e = 0.8970$ for the SMC-FOD. It can be seen that the performance of the SMC-FOD is better than that of the CSMC.

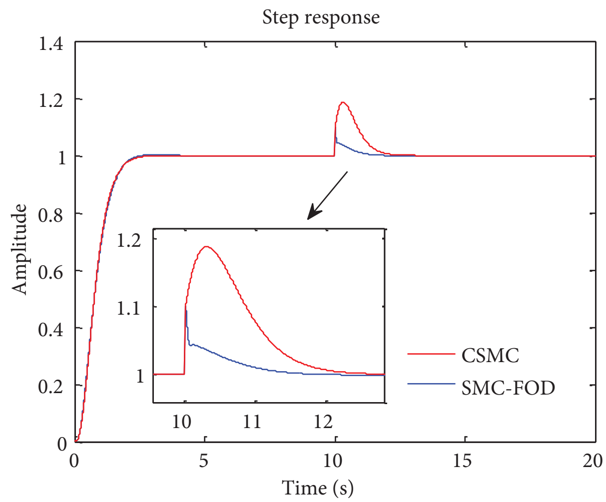

Figure 9. Step responses with output disturbance (noise) of 0.2 during 10-20 s.

4. Conclusion

This paper proposes to use FOD with the conventional SMC method to improve control performance. It is shown that the proposed SMC-FOD and CSMC are efficient to compensate for the effect of disturbances. However, the robustness of the system is much better with the proposed SMC-FOD method than with the conventional one with small dynamic tracking errors. It is clear from the simulation results that the SMC-FOD structure enlarges the span of the control efforts of the CSMC, which results in better control performance. The performance of the proposed method is demonstrated via the stability problem of unstable systems with time delay under output disturbances. The authors aim to design an adaptive SMC-FOD structure with optimized parameters to achieve better stability performance for more complicated plants in future works.

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