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BERNSTEIN POLYNOMIAL APPROACH AGAINST TO SOME FREQUENTLY USED GROWTH CURVE MODELS ON ANIMAL DATA

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ABSTRACT

Non-linear Logistic, Gompertz and Richards growth curve models were fitted to the data from Simmental x Southern Anatolian Red (SAR) crossbred cattles. Individual growth curves were fitted based on live weight measurements, and then general growth curves were obtained for all the models.

In addition, Bernstein basis polynomials have played important roles in nonparametric curve estimation. Therefore, Bernstein polynomial approach was used to model the growth curve in the current data. We determined the accuracy of the models by using coefficient of determination $\left(R^2\right)$, mean square error (MSE) and iteration number together.

In summary, the most suitable model based on the accuracy criteria was Bernstein model. Among the well-known growth curves, Logistic, Richards and Gompertz were ordered respectively.

KEY WORDS

Bernstein polynomial approach; Growth curve model; Simmental; Southern Anatolian Red.

1. INTRODUCTION

Body sizes of an individual change during growing. Because of the differential growth of the particular body parts, the shape of an organism (its proportions) changes as well. Unfortunately, one cannot measure continuously most of these growth processes. Therefore, it is preferable to model body measurements by mathematical growth functions. This gives one the opportunity to interpolate to non-observed intervals. Measurements of growth can be analyzed with respect to time (age) or to body weight using growth models (Şengül and Kiraz, 2005). Growth models can predict the biological characteristics of an individual, and parameters of growth models can describe lifetime growth. Also, growth models can determine optimum feeding regime, optimum age at slaughter and the effects of selection on growth curve parameters or body weight at any stage of life (Akbaş and Oğuz, 1998).

The use of Bernstein polynomial approach in addition to known methods in the estimation of dependent variable values is an important development in theory. At the

1912, S. Bernstein firstly introduced algebraic polynomial which approximates to a continuous function defined on closed interval via Weierstrass theorem. Bernstein basis polynomials have played important roles in nonparametric curve estimation. Bernstein polynomials are particular polynomials that form a basis for the space of polynomials. The value of the polynomial is then bounded by the values of the minimum and maximum Bernstein coefficients. The direct formula allows symbolic computation of these Bernstein coefficients giving a supplementary interest to the use of this theory. Another main interesting consequence is that the involved computations have quite low complexity. Moreover an appropriate instrumentation of this model allows automatic and inexpensive resolutions of complex program analysis issues (Tenbusch, 1997; Smith, 2009).

In the current study we intend to estimate general growth curve models of Simmental x SAR crossbred cattles using different non-linear growth functions. Also, Bernstein polynomial model was used to predict the values of dependent variable.

2. MATERIALS AND METHODS

The data were obtained from a project which was supported by The Scientific and Technological Research Council of Turkey (Project Number VHAG-950). The study was carried out at Ceylanpınar State Farms. SAR cows were reared in Ceylanpınar State Farm. A total of 52 female Simmental x SAR $F_1 x B_1$ crossbred was used in the current study. The birth weights of calves were taken before they were placed in the hutches. Live weights were taken monthly until the animals were 6 months of age. Then live weights were taken 90 days periods until 18 months of age. Some findings of this project were reported (Ertugrul et al. 1999).

We estimated individual growth curves using Gompertz, Logistic and Richards growth models for each cattle. Then, median values of the predicted parameters of the models were used to obtain general growth curves. Median values of the growth model parameters were substituted into the equation given below to estimate live weights in each model. The studied growth models and Bernstein polynomial model are as follows:

Gompertz : $f(x) = \alpha e^{\{-\beta e^{-\kappa x}\}}$,

Logistic : $y = \dfrac{\alpha}{1+\beta e^{-\kappa x}}$,

Richards : $y = \alpha[1+(\delta-1)\beta e^{-\kappa x}]^{1/1-\delta}$,

Bernstein : $\hat{y}(x) = \sum_{j=0}^{n} \sum_{k=0}^{n-j} C(n-k, j)\, C(n,k)\, y_k\, x^{n-j}$

where y is observed weight at age x, expressed in days, $\hat{y}(x)$ is predicted weight at age x, α, β, κ and δ are parameters of non-linear growth models (Seber and Wild, 1989). The criteria for the accuracy of the models used for the description of the growth were coefficient of determination $\left(R^2\right)$, mean square error (MSE) and iteration number together (Oliveira et al. 2000; Hassen et al. 2004). All calculations were performed using NCSS and SPSS. The related growth models can be obtained from the special solutions of the following differential equation:

$$z'(t) = g(z)\left[h(\alpha) - h(z)\right] \qquad (2.1)$$

where h and g are increasing functions which satisfy $g(0) = h(0) = 0$. Also $z'(t) \to 0$ must be satisfy when $t \to \infty$ and $t \to -\infty$. Gompertz model is achieved from the equation (2.1). While $h(z) = \ln z$ and $g(z) = c\,z$ are selected, the differential equation will be $z'(t) = c\,z(t)\,\ln\left(\alpha / z(t)\right)$, $c > 0$, $\alpha > 0$. The solution of this equation is $z(t) = \alpha\,\exp\left[-C\exp\left(-ct\right)\right]$, Gompertz model. Where $C \in R$ is the integral constant. The most general form of growth models is Richards. When $h(z) = z^{m-1}$, $g(z) = c\,(m-1)^{-1}\alpha^{\,1-m}z$ are opted, the equation will be $z'(t) = c\left(m-1\right)^{-1}\alpha^{\,1-m}z\left(\alpha^{\,m-1} - z^{m-1}\right)$, the solution of which is $z(t) = C^{ct/(m-1)}\left[\alpha^{\,m-1} - z^{m-1}\right]^{1/(m-1)}$, $m \neq 1$. This model can be arranged as $z(t) = \alpha\left[1 + C\exp\left(-ct\right)\right]^{1/(m-1)}$ (Seber and Wild, 1989).

3. RESULTS

Growth model parameters for each model are tabulated in Table 1. Table 2 summarizes the results of the criteria (the values of R^2, MSE and iteration number) for the accuracy of the models. All growth models and Bernstein model have considerably high (greater than 99%) values. If the growth models were arranged from greater to lesser R^2 values, we would obtain the following sequence: Bernstein polynomial model [0.999], Logistic [0.997], Richards [0.997] and Gompertz [0.994], respectively. α parameters of Logistic, Gompertz and Richards models were estimated as 540.429, 630 and 574.402, respectively.

The models may be ranked based on their MSE values as Bernstein polynomial model [30.97], Logistic [64.629], Richards [78.077] and Gompertz [110.593], respectively.

Among the models, the lowest iteration number was noted for Gompertz model, the other models from lesser to greater iteration number values were Logistic, Richards and Bernstein models, respectively.

The growth model parameters given in Table 1 were substituted into the equations given before to estimate live weights in each model as shown in Fig. 1. The predicted general growth curves estimated from median growth curve parameters of each model are given in Fig. 1. As can be seen in the figure, all growth curves show sigmoid growth patterns.

4. DISCUSSION

In the present study, our focus is not to obtain individual growth curves. The objective of the present study is to fit general growth curves using median growth curve parameters of each model. When considering Logistic, Gompertz and Richards models, κ parameters were similar. α and β parameters obtained from all models were different. Many studies (Oliveira et al. 2000; Hassen et al. 2004) have interpreted the biological significance of the parameter estimates of growth models. In contrast, we especially investigated the prediction performance of the studied growth models and Bernstein polynomial model based on the defined accuracy criteria in the current study.

Among all growth curve models, Logistic model seemed to be the best model describing the growth pattern regarding model selection criteria (i.e., coefficient of determination, mean square error and iteration number together). Richards model had higher R^2 and smaller MSE values than Gompertz model. Additionally when regarding performances of the Richards and Gompertz models, each model performed well in this study.

In addition, when compared Bernstein model with the growth curve models, Bernstein model is the fittest model based on the defined accuracy criteria.

The variations in the growth parameters of the models could be attributed to differences in the model properties. Generally, geographic location, different environmental conditions, species, age and sex can also affect the values of growth parameters (Hassen et al. 2004). Also, variation among breeds of cattle for production characteristics can be attributed to genetic and environmental causal components (Jenkins et al. 1991). Therefore, these conditions should be considered when the growth of different individuals is modeled.

Genetic potential of an animal may be explained by heritability estimates of growth curve parameters and phenotypic/genotypic correlations among growth curve parameters (Jenkins et al. 1991). For genetic selection purposes, it can be made use of heritability, phenotypic and genotypic correlations among growth curve parameters.

R^2 estimated for Logistic model is higher than those (0.91 and 0.90 based on pooled data) and is similar to that (0.99 based on individual animal regression) found by Hassen et al. (2004) for Angus bulls and heifer. Oliveira et al. (2000) obtained a growth pattern for 573 females Guzerat beef cattle through five non-linear models: Brody, Bertalanffy, Logistic, Gompertz, and Richard. Their R^2 values for the five models are smaller than those found in the present study for three models. Iteration number values in this study are higher for Logistic model than the value reported by Oliveira et al. (2000), and are smaller for Gompertz and Richard models than those reported by Oliveira et al. (2000). R^2 values in Gompertz, Logistic and Richard models reported by Behr et al. (2001) for Belgian Blue males/females cattle are smaller than the results achieved from this study. Enevoldsen and Kristensen (1997) developed seven regression models for data from 972 observations of 554 dairy cows. The R^2 values estimated in their study are smaller than the values in the current study.

Many studies (Quirino et al. 1999; Lambe et al. 2006) reported similar results using the models studied in this study or different models. The results of the studied models points out significant relationship between age and live weight. Previous studies (Perotto et al. 1992; Matthes et al. 1996) of growth curves in cattle reported that Richards was the most appropriate model fitted to data studied. However, based on the results of the present study, the best fitted model is Logistic, but Richards model also has a good performance in accordance with previous studies mentioned above. Similarly, Beltran et al. (1992) described and compared the growth patterns of Angus cows using Richards and Brody growth functions. The authors found that Brody model gave better estimates than Richards model. The findings of the reported studies suggest that each growth model may have different performance on the explanation of the growth. An alternative approach to growth curve models may be artificial neural networks (ANNs). Roush et al. (2006) compared Gompertz and neural network models of broiler growth. They reported that Gompertz growth model was fit for the data; nevertheless, ANNs produced the lowest bias, mean square error and mean absolute deviation.

The polynomial approach can be used to model the growth curve. Since the use of polynomial approach in growth curves has not frequently been used, this situation will be changed with the use of Bernstein polynomial approach in regression models (Bruce and Song, 1999). The application of Bernstein polynomial approach in regression modeling is so difficult owing to need of computing time and memory which grows exponentially with the number of variables. However, this approach gives good results in the estimation of dependent variable values. But, large sample sizes are required to minimize mean square error in this approach. In the application of Bernstein model in the current data, we estimated the smallest mean square error and the highest R^2 values. Some advantages of Bernstein model for the analysis of multivariate polynomials are that it generally more accurate than classic interval methods, and it handles any multivariate polynomial without restriction (Delgado and Pena, 2003). Gürcan and Çolak (2009) investigated the well-known growth curves together with Bernstein polynomial functions and concluded that Bernstein polynomial model gives lower error than the well-known growth curves.

Finally, the most suitable model based on the accuracy criteria was Bernstein model. Among the well-known growth curves, Logistic, Richards and Gompertz were ordered respectively.

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Table 1:
Growth model parameters for Simmental x SAR crossbred cattles

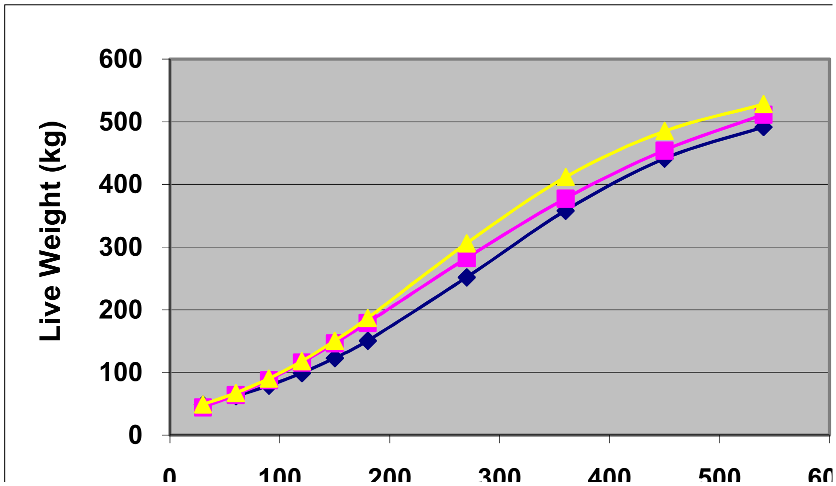
		F_1xG_1 (n=52)	
Model	**Parameter**	X_{50}	$S_{X_{50}}$
Logistic	α	540.429	20.908
	β	13.224	0.583
	κ	0.009	2.788E-04
Gompertz	α	630	18.476
	β	3.089	0.060
	κ	0.004	1.425E-04
Richards	α	574.402	11.982
	β	6.499	1.131
	κ	0.008	4.266E-04
	δ	1.532	0.096

X_{50} : Median value, $S_{X_{50}}$: Standard Error (SE) of estimates; α , β , κ and δ are model parameters.

Table 2:
Results of the criteria for the accuracy of the models

Criteria	**Model**	X_{50}
MSE	Logistic	64.629
	Gompertz	110.593
	Richards	78.077
R^2	Logistic	0.997
	Gompertz	0.994
	Richards	0.997
Iteration Number	Logistic	13
	Gompertz	6
	Richards	15
Criteria	**Model**	**Estimation**
MSE	Bernstein	30.97
R^2		0.999
Iteration Number		32

X_{50} : Median value; MSE: Mean Square Error; R^2 : coefficient of determination



L: Logistic, G: Gompertz, R: Richards

Fig. 1: **Growth Curves for Simmental *x* SAR crossbred cattles**